Name:

Date:

Start Date of Calc I:

Project 2: Solution Critique

Directions:

1. Analyze the solutions presented to various integration problems provided below. State whether each solution is correct or incorrect. If the solution is incorrect, state where the solution is incorrect and provide a fully worked correct solution.
2. **Type your answers in BOLD using GREEN colored font next to each question.**
3. Insert photos/tables directly into the document, as directed.

    1. Using a scanner or a free app (like CamScanner) select the option that converts your photos into “Black and White” or “Black and White with Color” **pdf**. This will provide clarity in reading your solutions. Your insertions **should not look like a photo** when submitted.

    2. When using a free pdf converter app, many free apps include a watermark of the software. Make sure that the **watermark does not cover** any of your solutions. If your solution cannot be read, it will not receive credit.

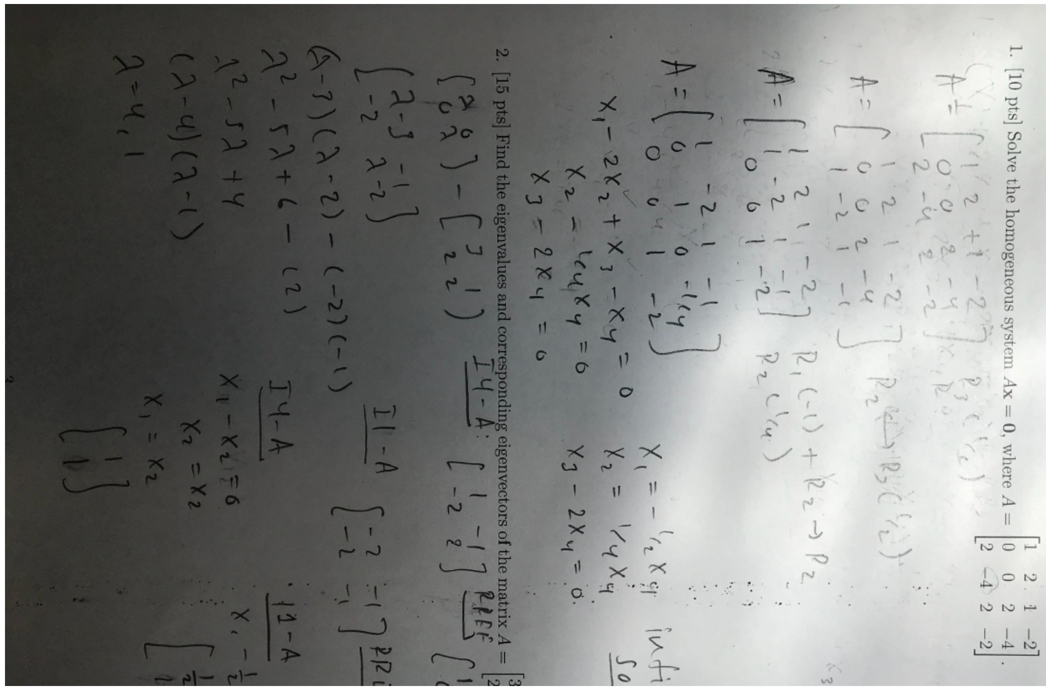
    3. Your file must be **clear and not fuzzy**. Solutions that are not clear will not receive credit.

    4. **Crop your insertion** to only include your work. The insertion should not contain other items, such as your table top or computer.

    5. Insertions should be **orientated facing up** and not upside down or sideways. The photo should show all of your solutions.

    6. Students are expected to clearly communicate their solutions in a **professional and polished manner**. Below is an example of an unacceptable insertion and a properly submitted insertion. Improperly submitted insertions will not receive credit.

        1. Unacceptable Insertion



ii. Proper Insertion

3)

$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ K & 0 & 1 \end{bmatrix}$ $KR_1 + R_3 \rightarrow R_3$ $\updownarrow$ $(-K)R_1 + R_3 \rightarrow R_3$

$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -K & 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $R_1 \leftrightarrow R_2$ $\updownarrow$ $R_2 \leftrightarrow R_1$

$E_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & K \end{bmatrix}$ $KR_3 \rightarrow R_3$ $\updownarrow$ $\frac{1}{K} R_3 \rightarrow R_3$

$E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{K} \end{bmatrix}$

4) $A = \begin{bmatrix} 4 & -1 \\ 3 & -1 \end{bmatrix}$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

① $R_1 + (-1)R_2 \rightarrow R_1$

$\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

② $(-3)R_1 + R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

③ $(-1)R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$I = E_3 E_2 E_1 A$

↑

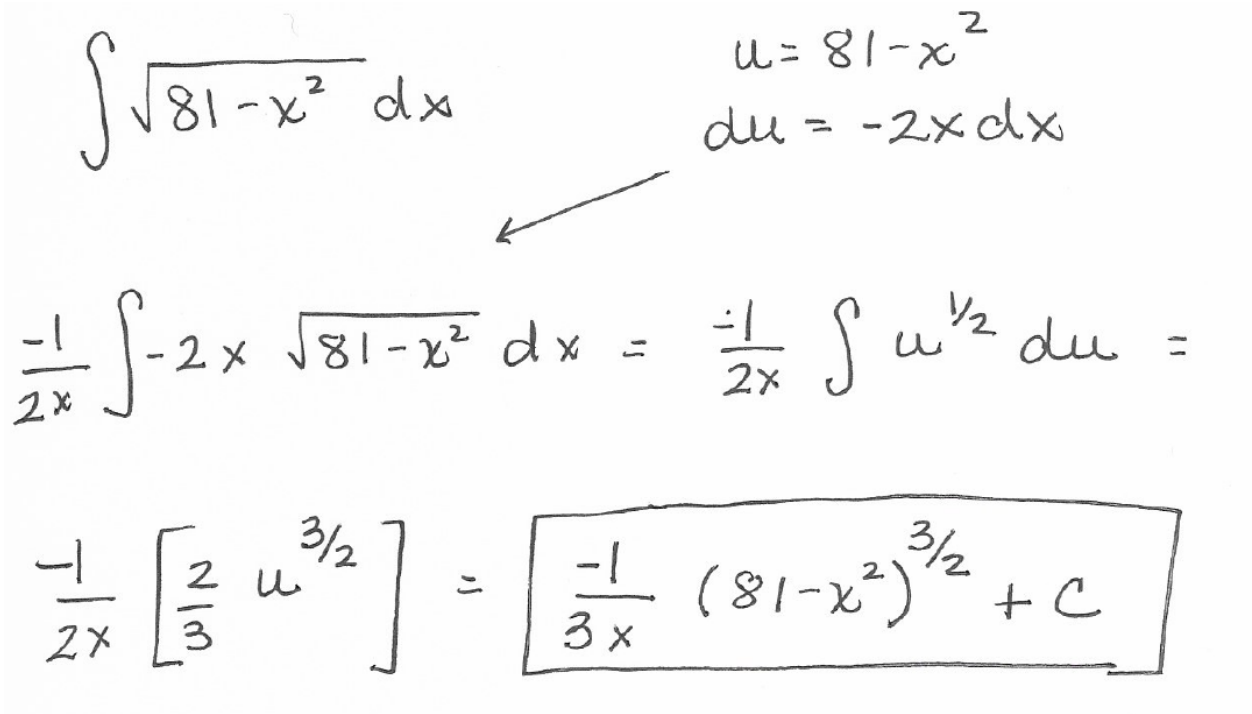
1. Upload this completed document to Blackboard.

Solutions to Critique

Question 1: Find the most general antiderivative to the following.

$$\int \sqrt{81 - x^2}\, dx$$

Question 1 Solution:



Response to Question 1 Solution:

Solution is: Correct/ Incorrect (highlight one)

Critique: (In a few sentences explain the errors in the above provided solution , if any. Use an orange colored font.)

(Insert corrected solution here. Your solution should contain all necessary steps of the integration and an explanation of the solution process.)

Question 2: Draw the region bounded by the following curves. Find the volume when the region is rotated around the x-axis.

$$y = \sqrt{1 + x^2} \quad \text{and} \quad y = \sqrt{4 - x^2}$$

Question 2 Solution:

Find intersection points:

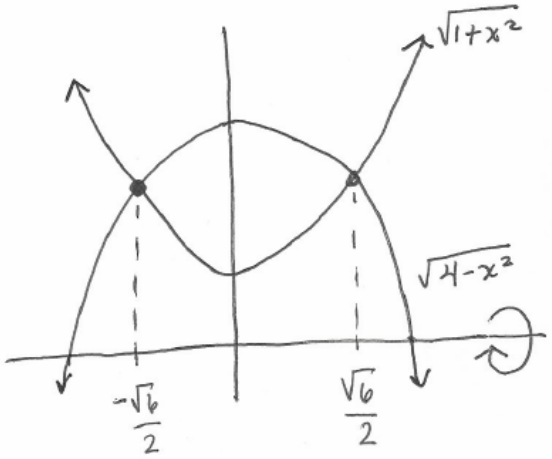
$$\sqrt{1+x^2} = \sqrt{4-x^2}$$

$$1 + x^2 = 4 - x^2$$

$$2x^2 = 3$$

$$x^2 = 3/2$$

$$x = \pm\sqrt{3/2} = \pm\frac{\sqrt{6}}{2}$$



$$V = \int_a^b \pi\left[f(x)^2 - g(x)^2\right]dx = \pi\int_{-\sqrt{6}/2}^{\sqrt{6}/2} \left(\sqrt{4-x^2}\right)^2 - \left(\sqrt{1+x^2}\right)^2 dx$$

$$= 2\pi\int_0^{\sqrt{6}/2} 4 - x^2 - 1 - x^2\,dx = 2\pi\int_0^{\sqrt{6}/2} 3 - 2x^2\,dx = 2\pi\left[3x - \frac{2x^3}{3}\Big|_0^{\sqrt{6}/2}\right]$$

$$= 2\pi\left[3\left(\frac{\sqrt{6}}{2}\right) - \frac{2}{3}\left(\frac{\sqrt{6}}{2}\right)^3\right] = 2\pi\left[\frac{3\sqrt{6}}{2} - \frac{2}{3}\cdot\frac{6}{4}\right] = 2\pi\left[\frac{3\sqrt{6}}{2} - 1\right]$$

$$= \boxed{3\pi\sqrt{6} - 2\pi}$$

Response to Question 2 Solution:

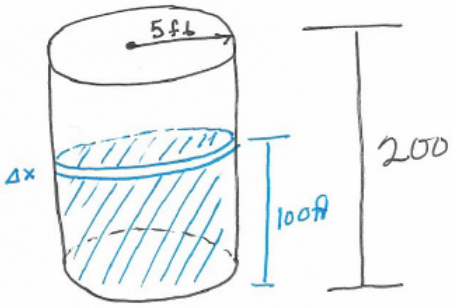
Solution is: Correct/ Incorrect (highlight one)

Critique: (In a few sentences explain the errors in the above provided solution , if any. Use an orange colored font.)

(Insert corrected solution here. Your solution should contain all necessary steps of the integration and an explanation of the solution process.)

Question 3: Find the work required to pump all the water out of a half full cylinder that has a circular base of radius 5 ft and height of 200 ft. Use water density = 62 lb/ft^3

Question 3 Solution:



Volume of i^{th} disk: $\pi(5)^2 \Delta x = 25\pi \Delta x$

Force to lift each layer: $62(25\pi \Delta x) = 1550\pi \Delta x$

Work for each layer: $1550\pi x_i \Delta x$

$$\text{Work}_{\text{Total}} \approx \sum_{i=1}^{n} 1550\pi x_i \Delta x$$

$$W_{\text{Total}} = \lim_{n\to\infty} \sum_{i=1}^{n} 1550\pi x_i \Delta x = \int_0^{100} 1550\pi x \, dx =$$

$$\frac{1550\pi}{2}\left[x^2\Big|_0^{100}\right] = 775\pi(100^2) = \boxed{7{,}750{,}000\pi \text{ Joules}}$$

Response to Question 3 Solution:

Solution is: Correct/ Incorrect (highlight one)

Critique: (In a few sentences explain the errors in the above provided solution , if any. Use an orange colored font.)

(Insert corrected solution here. Your solution should contain all necessary steps of the integration and an explanation of the solution process.)